Estimating Simulation Parameters

Reconciling Monte Carlo Simulation and The Black-Scholes Model

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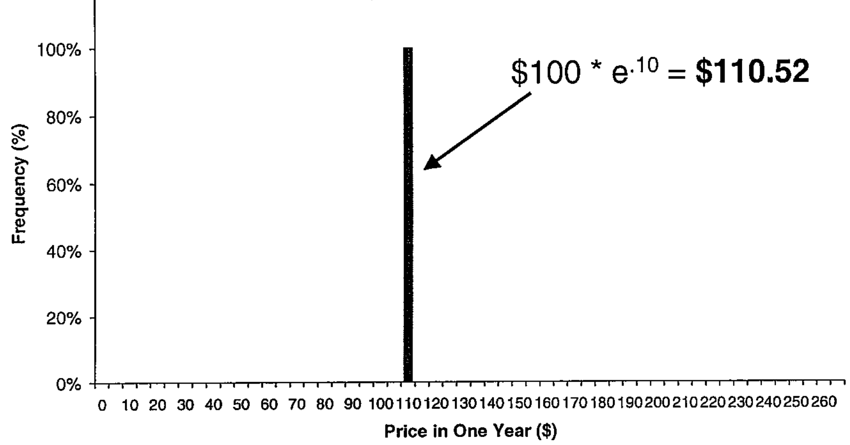
Estimating the Interest Rate in Monte-Carlo Simulations

- To price an option using the Black-Scholes option pricing formula, one must define a number of input parameters:
  - S_0 – the current stock price
  - K – the strike price (at which the option is struck)
  - T – the time to expiration in years
  - σ – stock price volatility
  - μ – the continuously compounded risk-free rate
- More specifically, μ is the expected continuously compounded risk-free return earned by an investor per year
- A European call option can similarly be priced using simulation by the following method:
  - Simulate a lognormal return of the stock
  - For each simulation path, take the payoff of the option
  - Discount the payoff using the risk-free rate
  - The expected value of the PV of the payoff is the value of the option
- In a Black-Scholes world all returns are equal to the risk-free return
  - This implies that the expected value of a risk-free zero coupon bond in time 1 equals the expected value of a stock return that has stochastic properties

Illustrative Simulation Results

Risk-free Zero-Coupon T-Bill

$\mu = 10\%$



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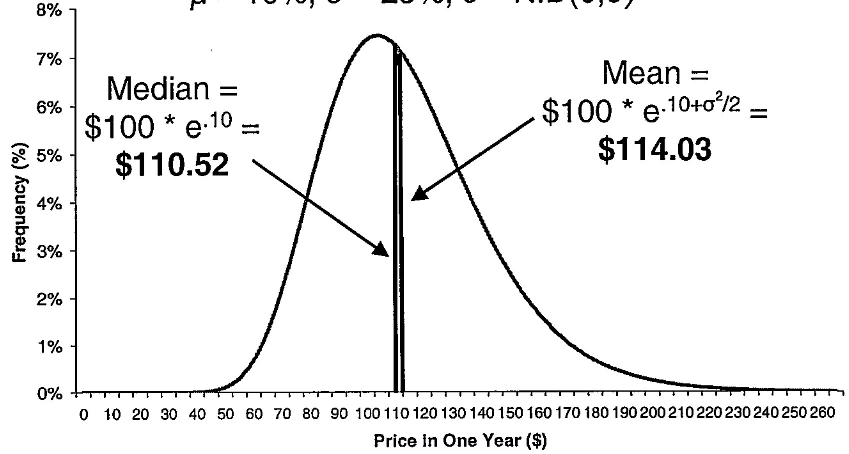
Simulation: $\ln S_{t+1} = \ln S_t + \mu + \varepsilon_{t+1}$

$\mu = 10\%$; $\sigma = 25\%$; $\varepsilon \sim NID(0,\sigma)$



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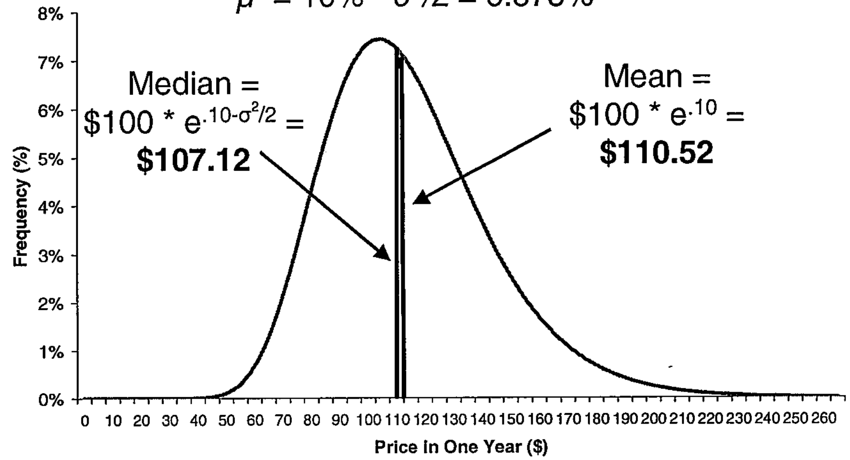
- Intuitively, the expected return on a risk-free investment (e.g. a zero-coupon T-bill) at time 1 has to equal the expected value of the stochastic stock return at time 1
- This can be achieved by setting the drift in the simulation to $\mu' = \mu - \sigma^2 / 2$

Simulation: $\ln S_{t+1} = \ln S_t + \mu' + \varepsilon_{t+1}$

$\mu' = 10\% - \sigma^2/2 = 6.875\%$



Black-Scholes vs. Simulation Example

- Assume the following parameters to price a European call option
  - S_0 – the current stock price = $100
  - K – the strike price = $100
  - T – the time to expiration in years = 1 year
  - σ – stock price volatility = 25%
  - μ – the continuously compounded risk-free rate = 10%
- The Black-Scholes option value based on the above parameters is $14.98
- A simulation using $\mu' = \mu - \sigma^2 / 2 = 0.1 - 0.25^2 / 2 = 6.875\%$ would result in a lognormal distribution with an average stock price, after one year, of $110.52
  - This is equivalent of a 10% continuously compounded appreciation (i.e. $e^{.10} = 1.1052$)
- Calculating the payoff for the call option and then discounting it by a continuously compounded rate of 10% (or a simple rate of 10.52%) results in an average value for the option of $14.98